Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MPC3

Unit Pure Core 3

Wednesday 23 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Show that the equation $x^3 - 6x + 1 = 0$ has a root α, where $2 < \alpha < 3$. *(2 marks)*

(b) Show that the equation $x^3 - 6x + 1 = 0$ can be rearranged into the form

$$x^2 = 6 - \frac{1}{x}$$ *(1 mark)*

(c) Use the recurrence relation $x_{n+1} = \sqrt{6 - \frac{1}{x_n}}$, with $x_1 = 2.5$, to find the value of x_3, giving your answer to four significant figures. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

2 (a) Use Simpson's rule, with five ordinates (four strips), to calculate an estimate for

$$\int_0^4 \frac{x}{x^2+2}\,\mathrm{d}x$$

Give your answer to four significant figures. *(4 marks)*

(b) Show that the exact value of $\displaystyle\int_0^4 \frac{x}{x^2+2}\,\mathrm{d}x$ is $\ln k$, where k is an integer. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 2

3 (a) Find $\dfrac{dy}{dx}$ when

$$y = e^{3x} + \ln x$$

(2 marks)

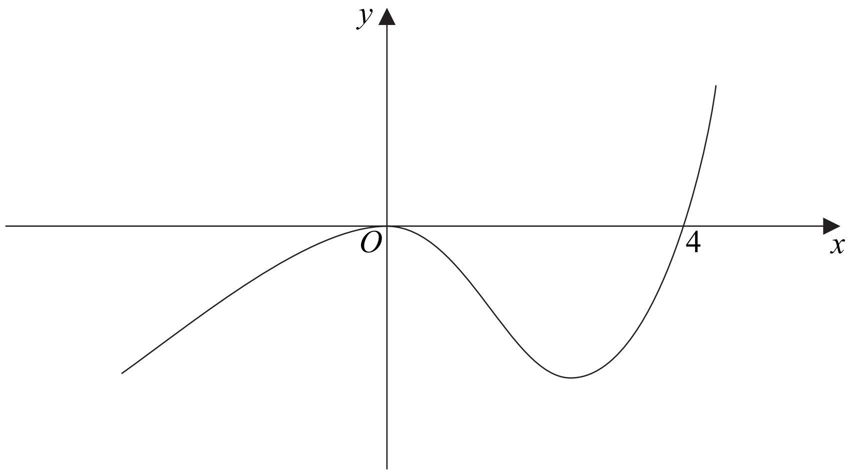
(b) (i) Given that $u = \dfrac{\sin x}{1 + \cos x}$, show that $\dfrac{du}{dx} = \dfrac{1}{1 + \cos x}$. *(3 marks)*

(ii) Hence show that if $y = \ln\left(\dfrac{\sin x}{1 + \cos x}\right)$, then $\dfrac{dy}{dx} = \operatorname{cosec} x$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

4 The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$.



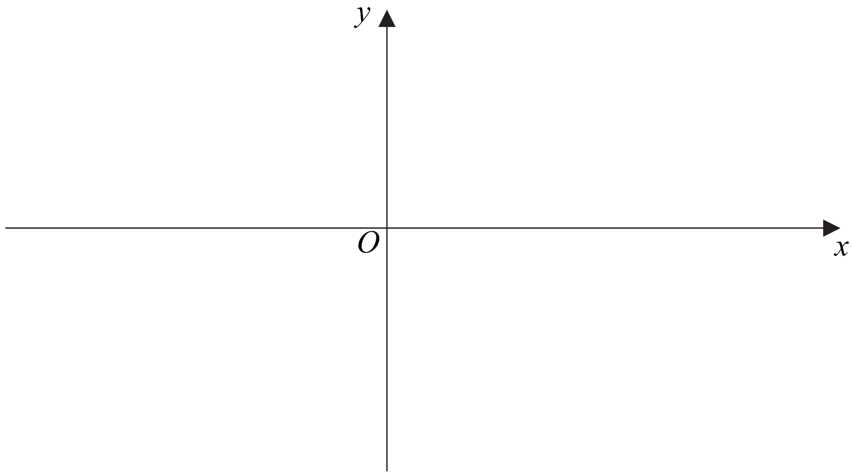
(a) On the axes below, sketch the curve with equation $y = |\mathrm{f}(x)|$. *(2 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(2x - 1)$. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 4

(a)



5 The function f is defined by

$$f(x) = \frac{x^2 - 4}{3}, \quad \text{for real values of } x, \textbf{ where } \boldsymbol{x \leqslant 0}$$

(a) State the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

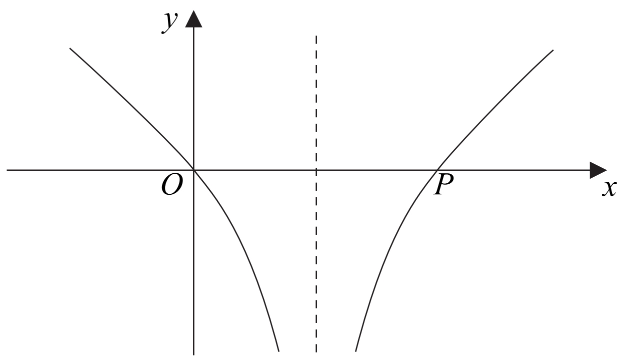
(i) Write down the domain of f^{-1}. *(1 mark)*

(ii) Find an expression for $f^{-1}(x)$. *(3 marks)*

(c) The function g is defined by

$$g(x) = \ln|3x - 1|, \quad \text{for real values of } x, \text{ where } x \neq \frac{1}{3}$$

The curve with equation $y = g(x)$ is sketched below.



(i) The curve $y = g(x)$ intersects the x-axis at the origin and at the point P.

Find the x-coordinate of P. *(2 marks)*

(ii) State whether the function g has an inverse. Give a reason for your answer. *(1 mark)*

(iii) Show that $gf(x) = \ln|x^2 - k|$, stating the value of the constant k. *(2 marks)*

(iv) Solve the equation $gf(x) = 0$. *(4 marks)*

6 (a) Show that

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)}$$

can be written as $\operatorname{cosec}^2 x$. *(3 marks)*

(b) Hence solve the equation

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)} = \operatorname{cosec} x + 3$$

giving the values of x to the nearest degree in the interval $-180° < x < 180°$. *(6 marks)*

(c) Hence solve the equation

$$\frac{\sec^2(2\theta - 60°)}{(\sec(2\theta - 60°) + 1)(\sec(2\theta - 60°) - 1)} = \operatorname{cosec}(2\theta - 60°) + 3$$

giving the values of θ to the nearest degree in the interval $0° < \theta < 90°$. *(2 marks)*

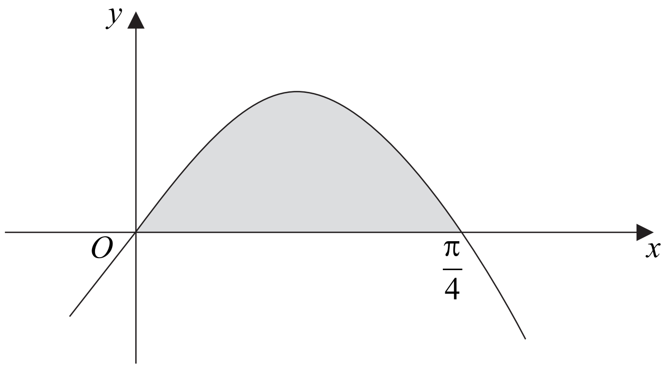
QUESTION PART REFERENCE

Answer space for question 6

7 A curve has equation $y = 4x\cos 2x$.

(a) Find an exact equation of the tangent to the curve at the point on the curve where $x = \dfrac{\pi}{4}$. *(5 marks)*

(b) The region shaded on the diagram below is bounded by the curve $y = 4x\cos 2x$ and the x-axis from $x = 0$ to $x = \dfrac{\pi}{4}$.



By using integration by parts, find the exact value of the area of the shaded region. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 7

8 (a) Show that

$$\int_0^{\ln 2} e^{1-2x}\,dx = \frac{3}{8}e$$

(4 marks)

(b) Use the substitution $u = \tan x$ to find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^4 x\sqrt{\tan x}\,dx$$

(8 marks)

QUESTION PART REFERENCE

Answer space for question 8